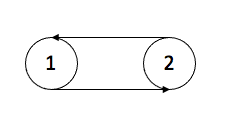
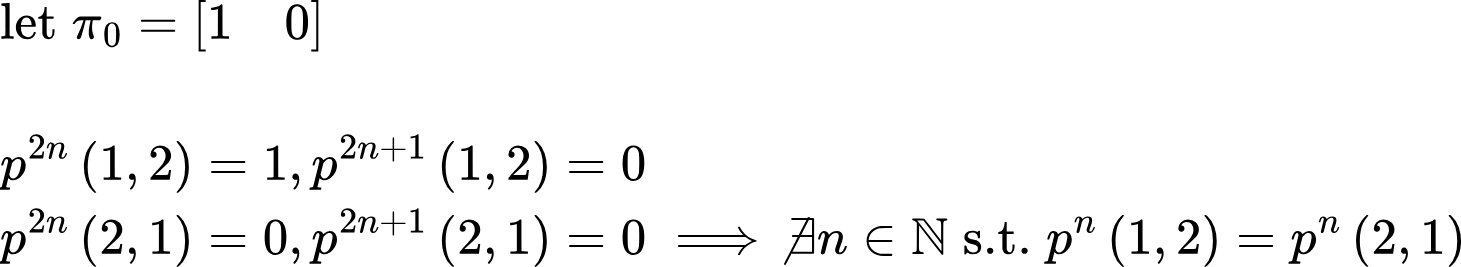
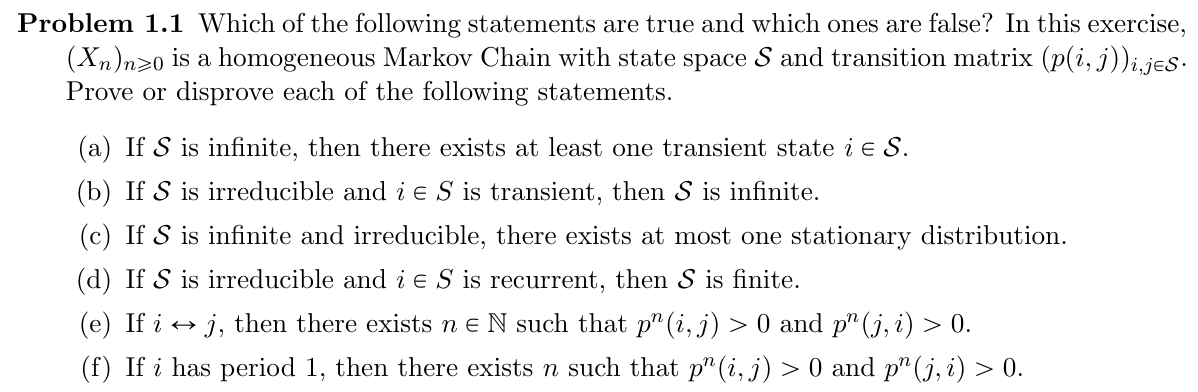
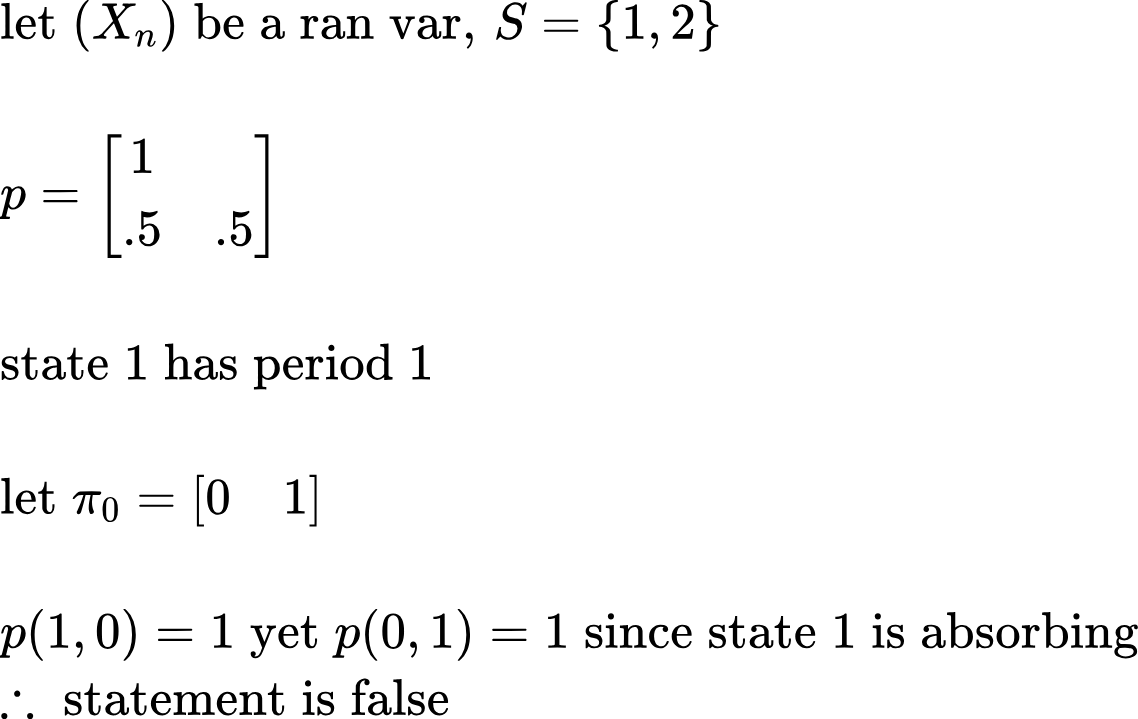


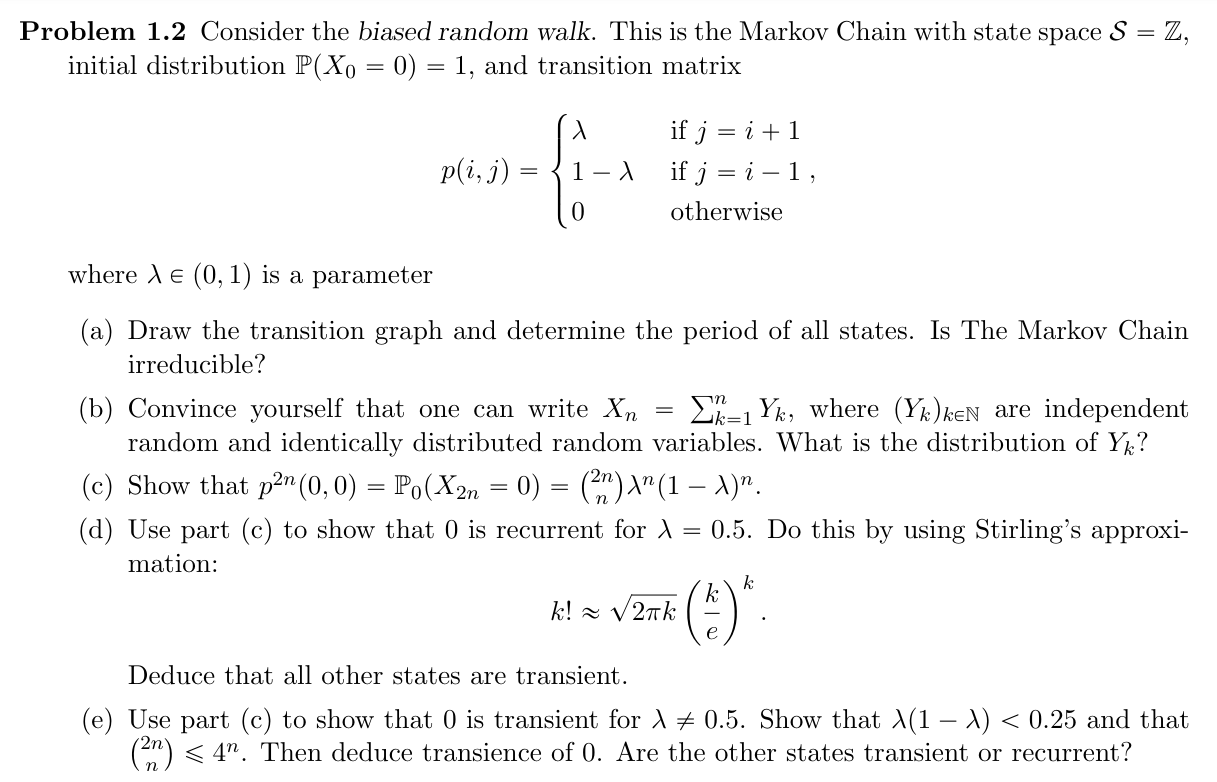
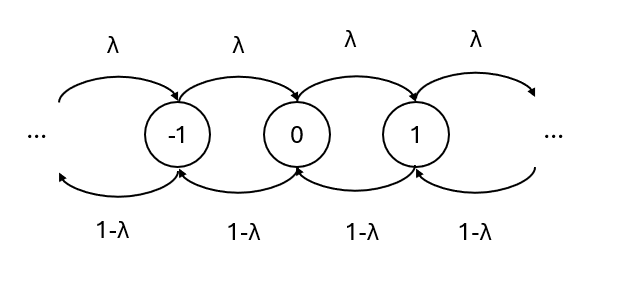
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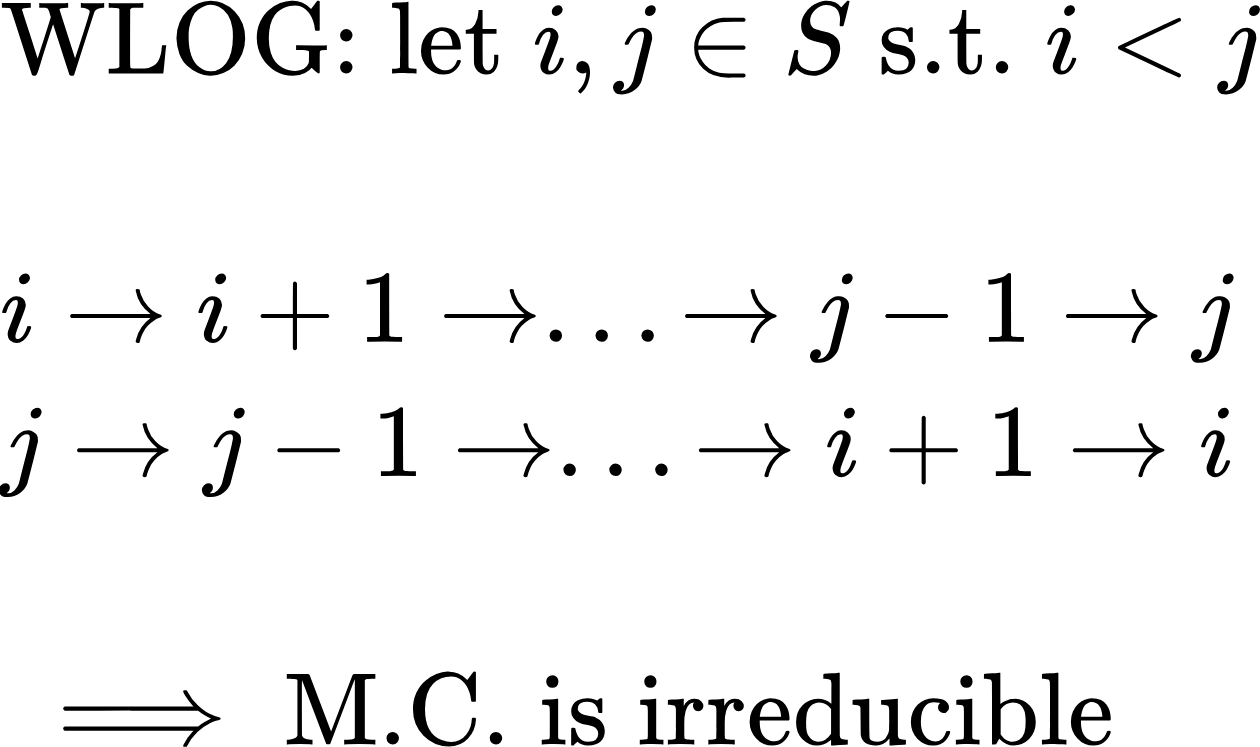


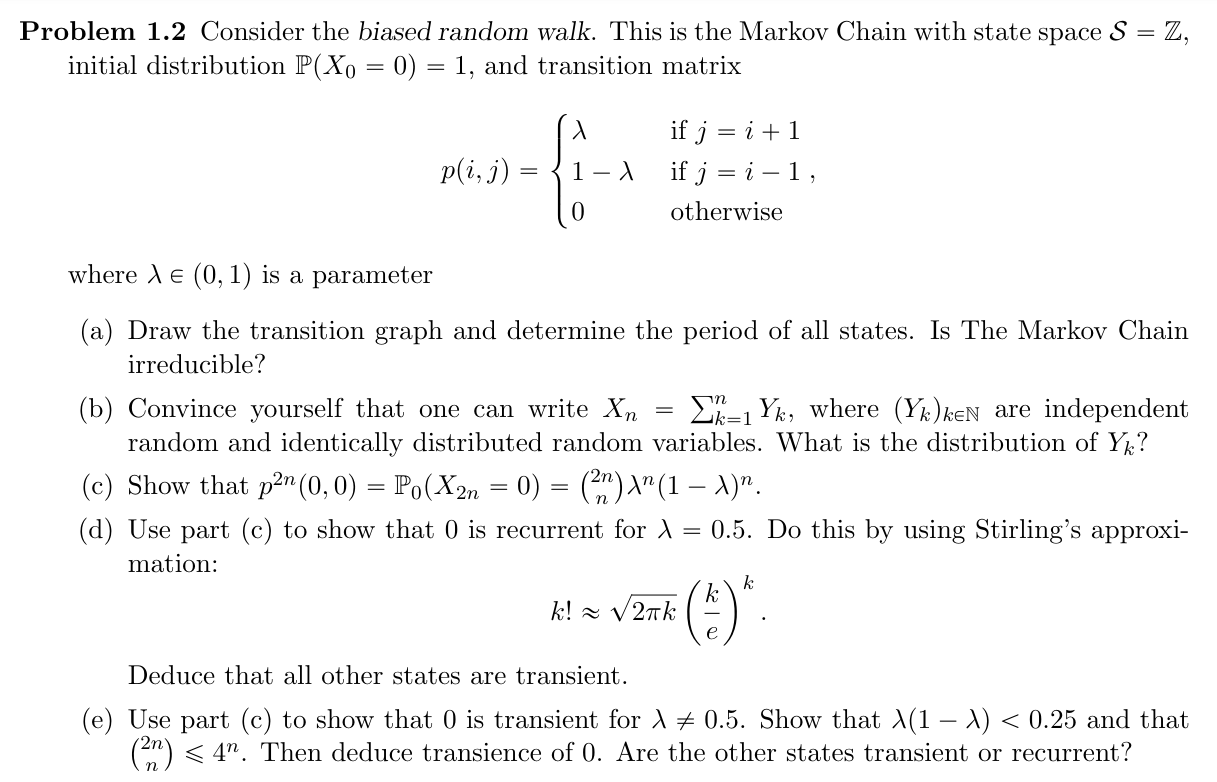


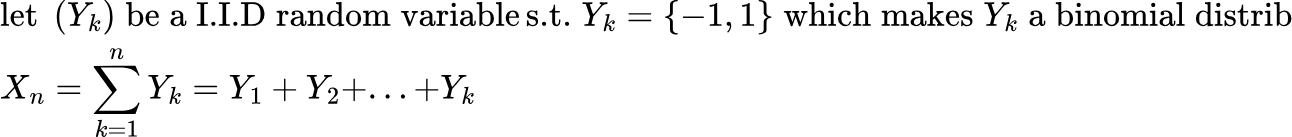


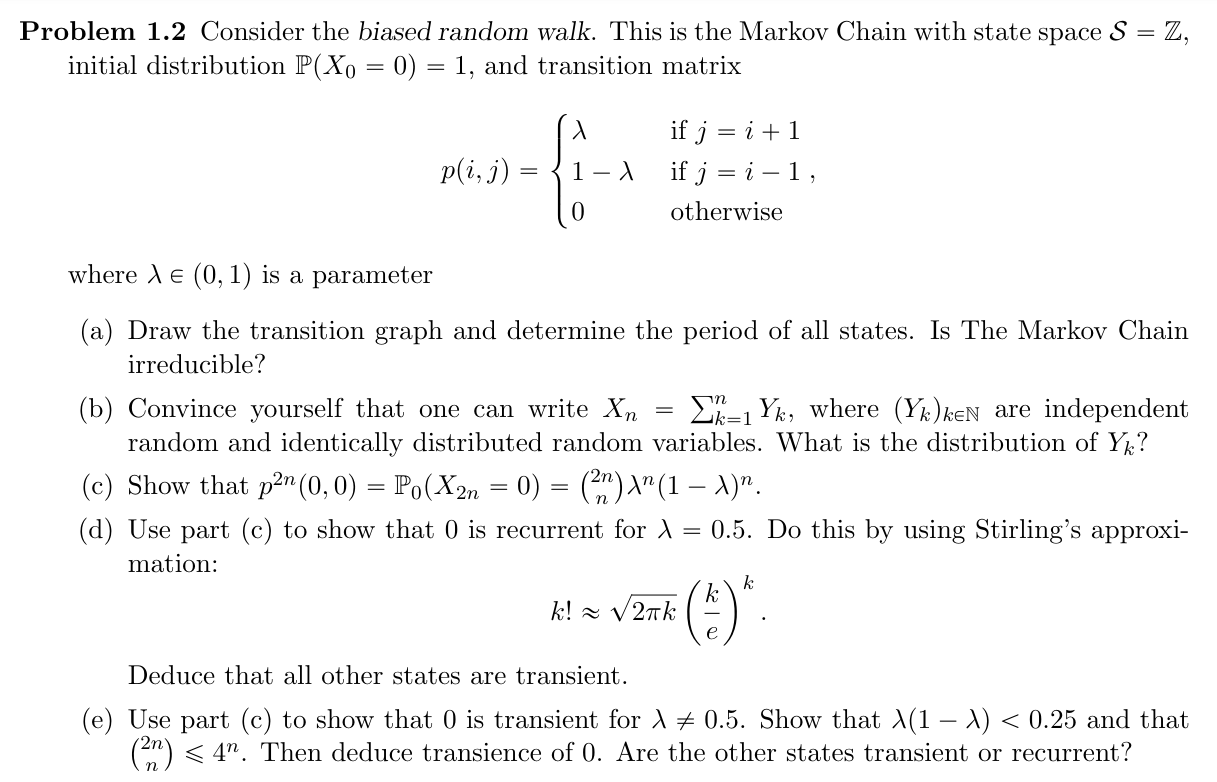


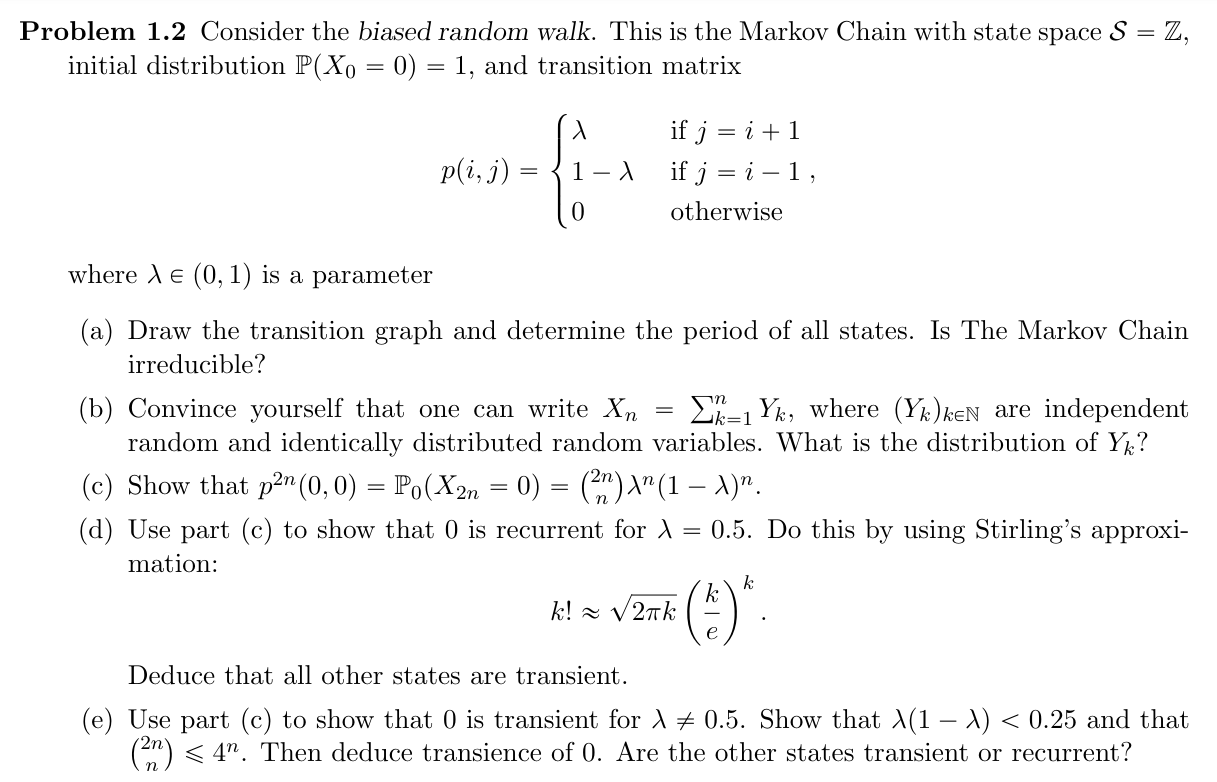




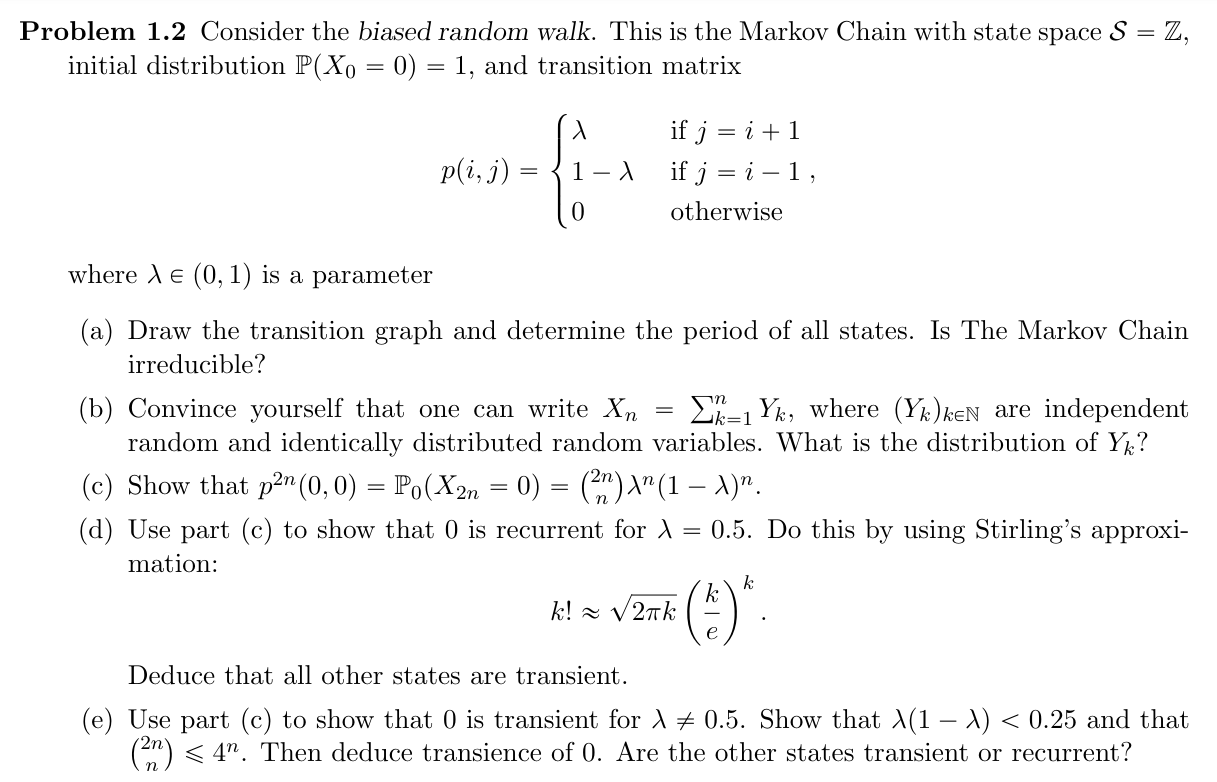


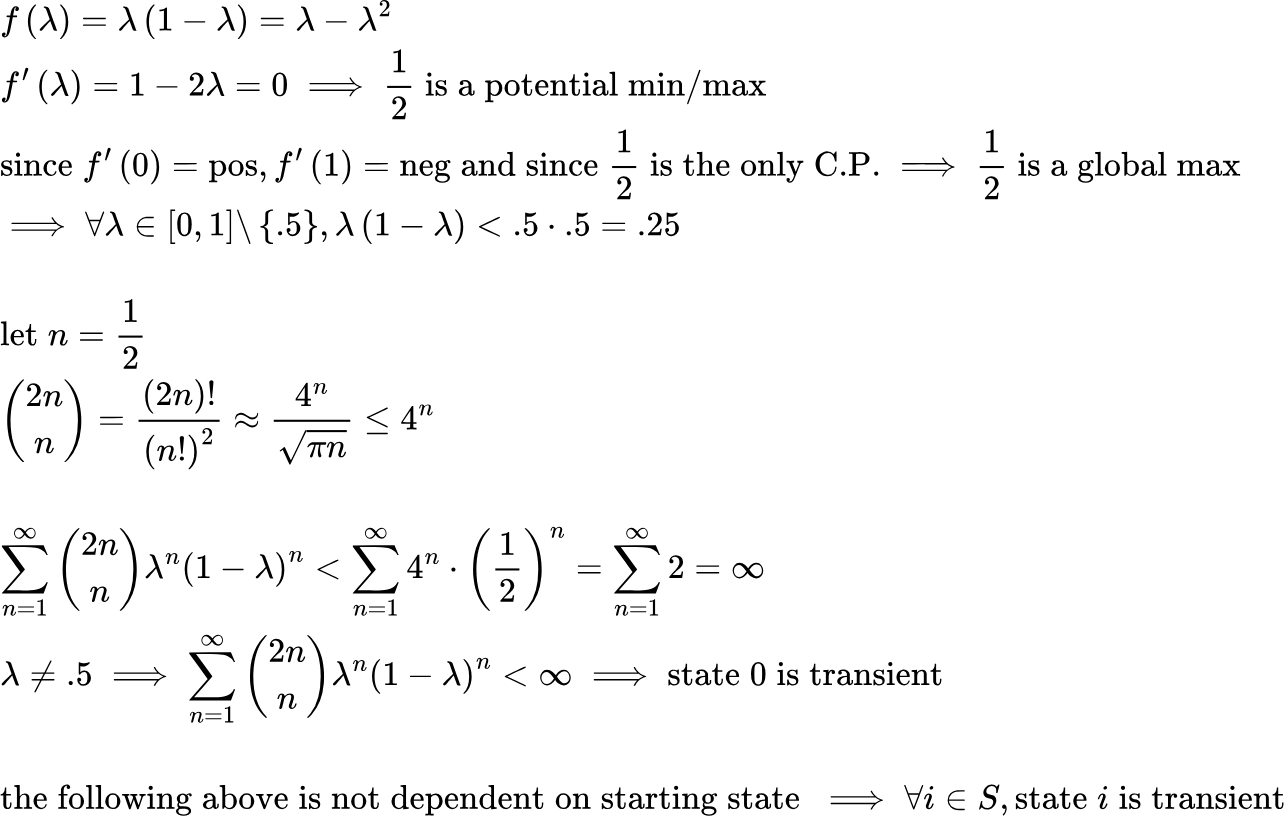


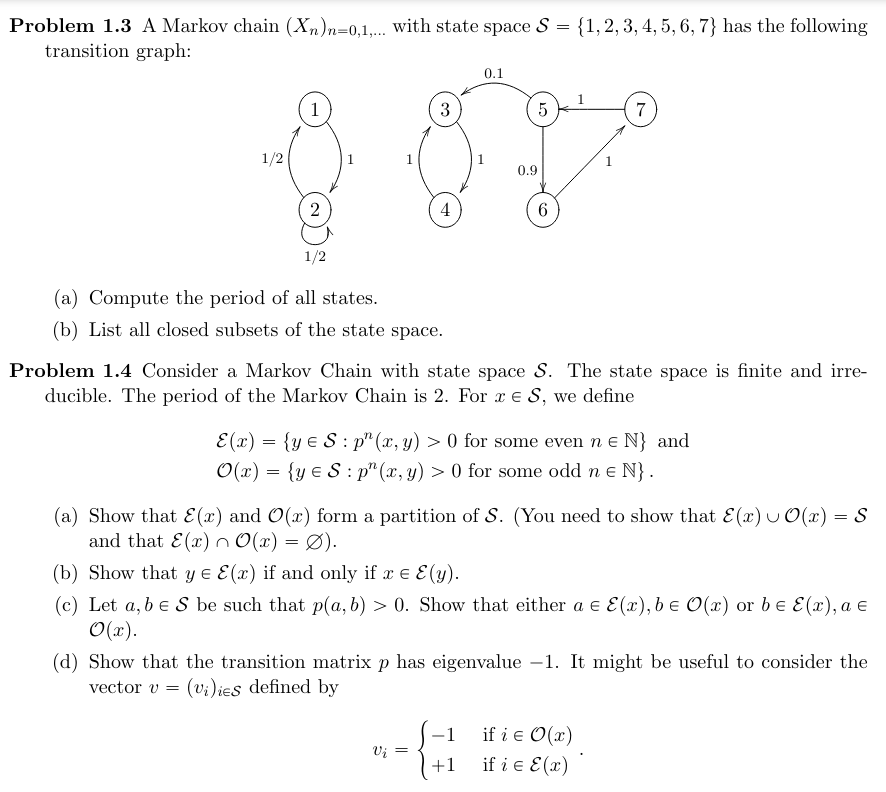
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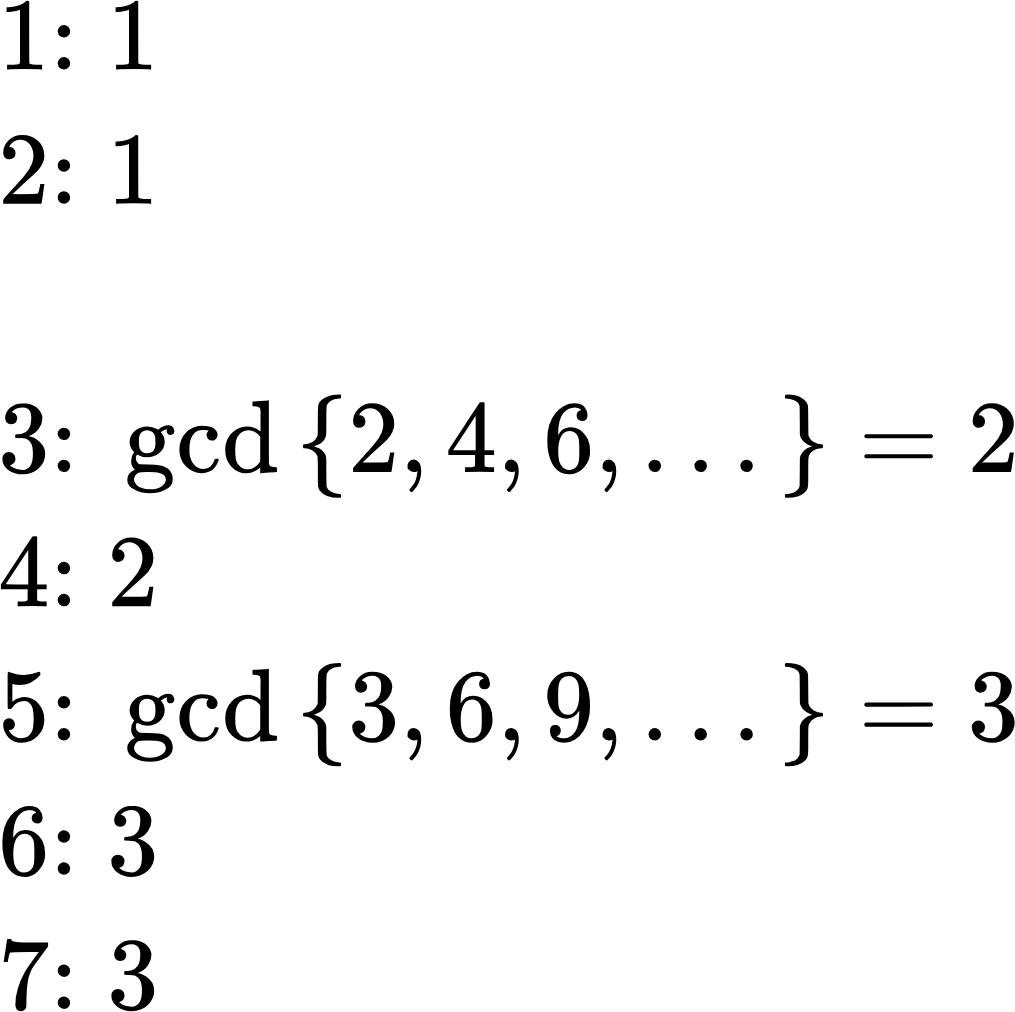


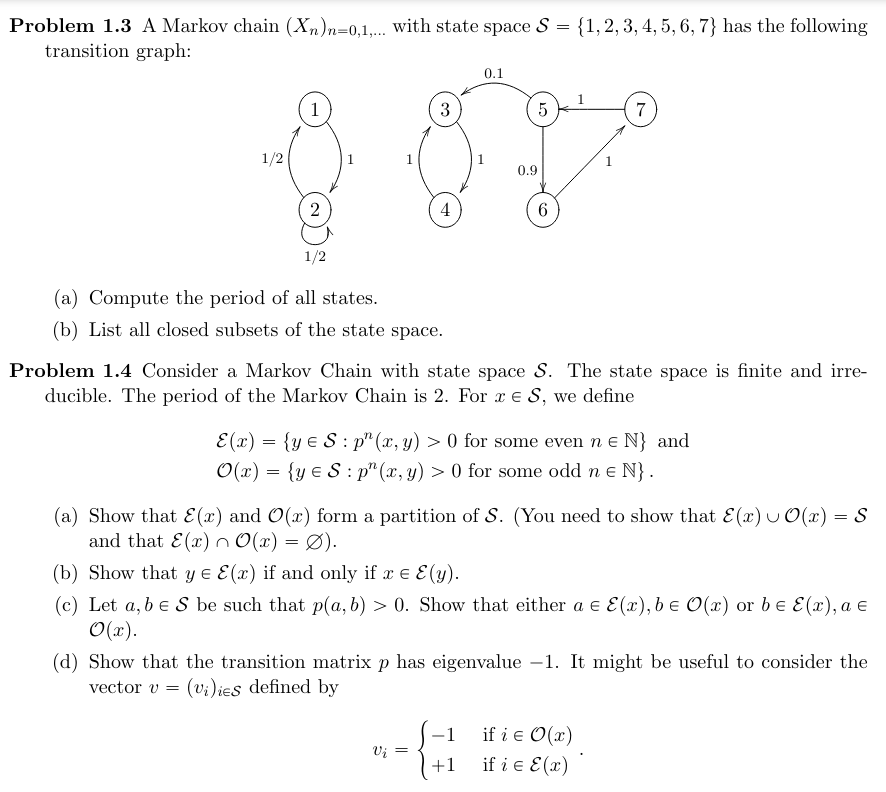
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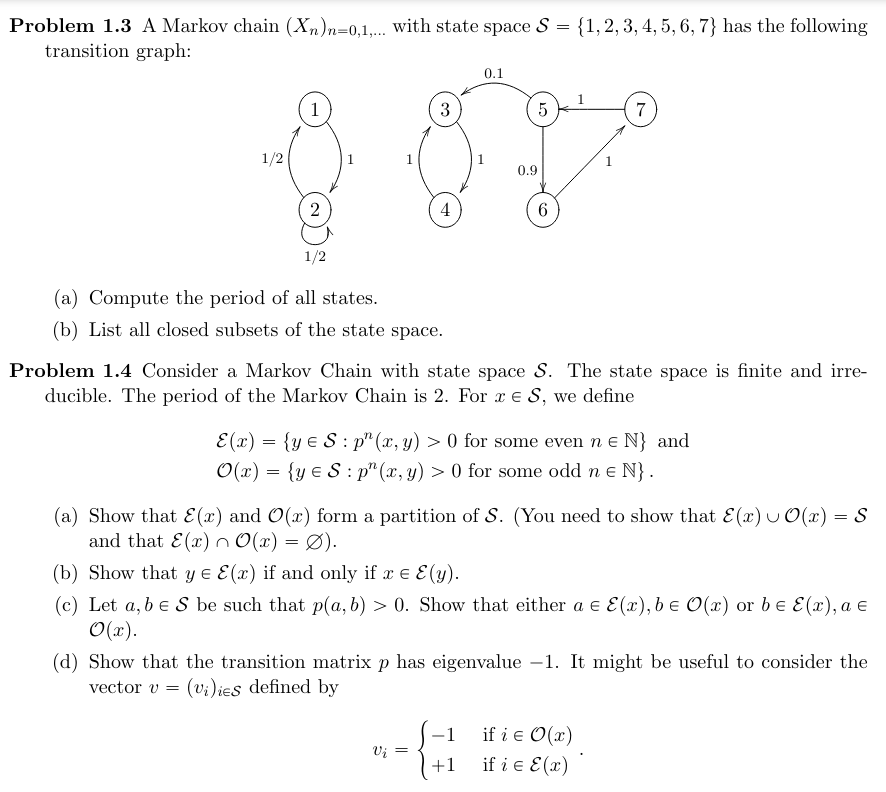


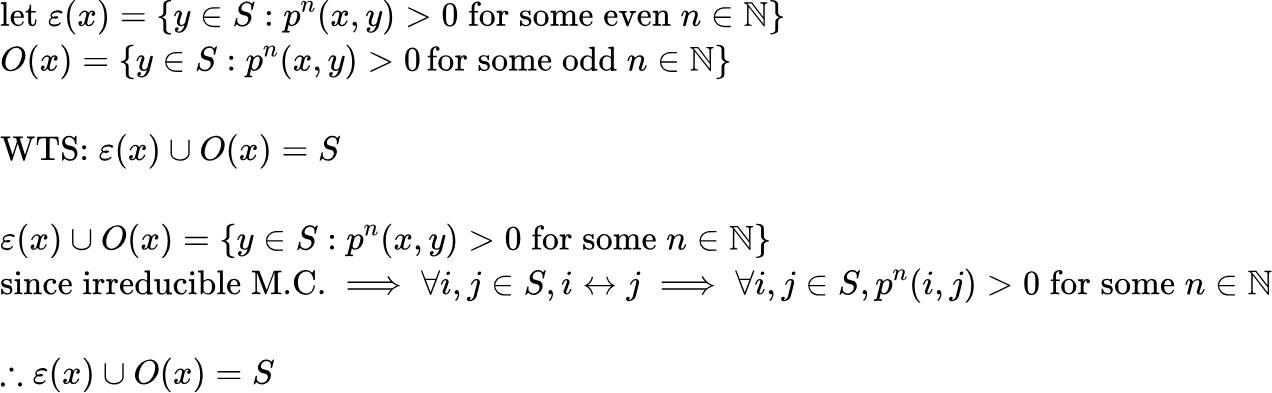


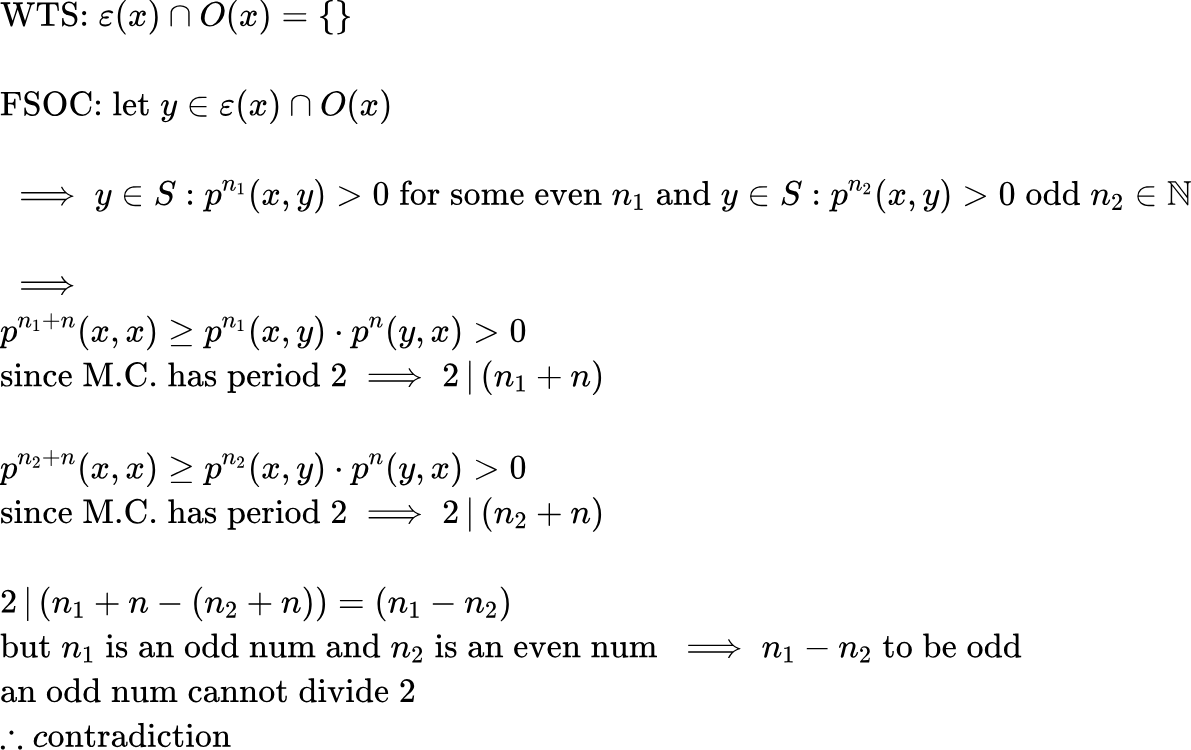


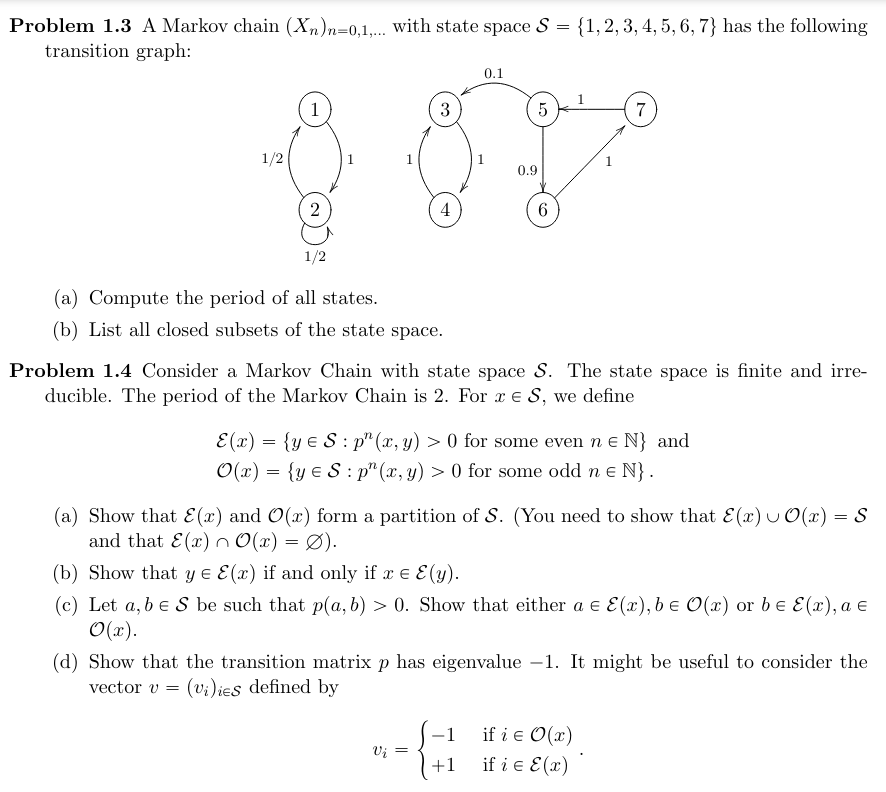


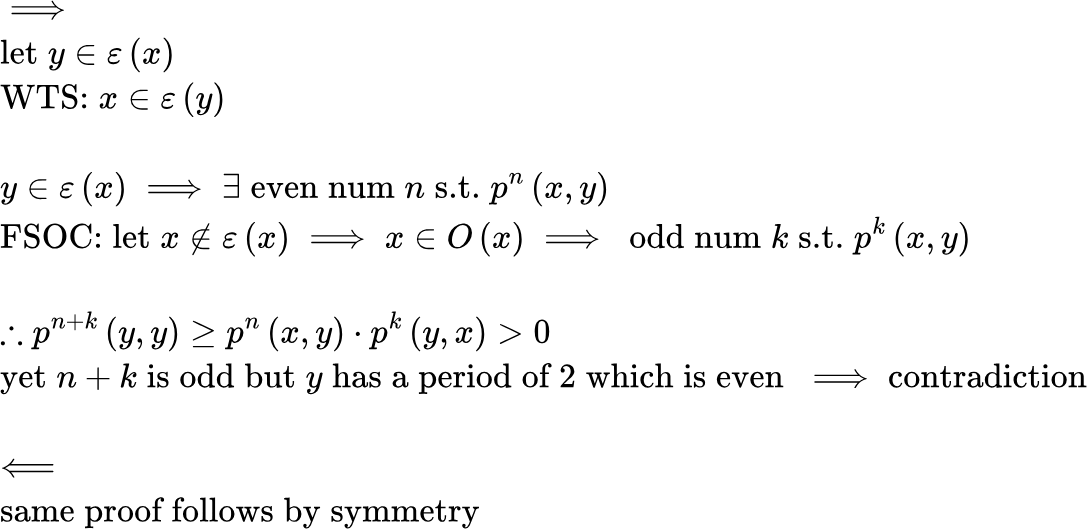
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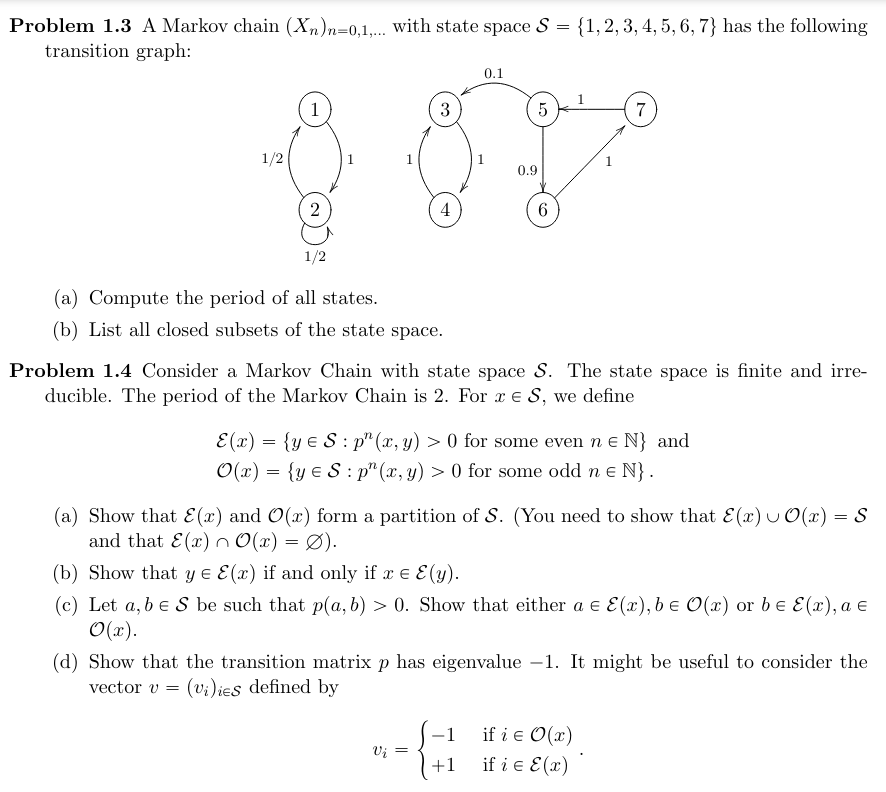


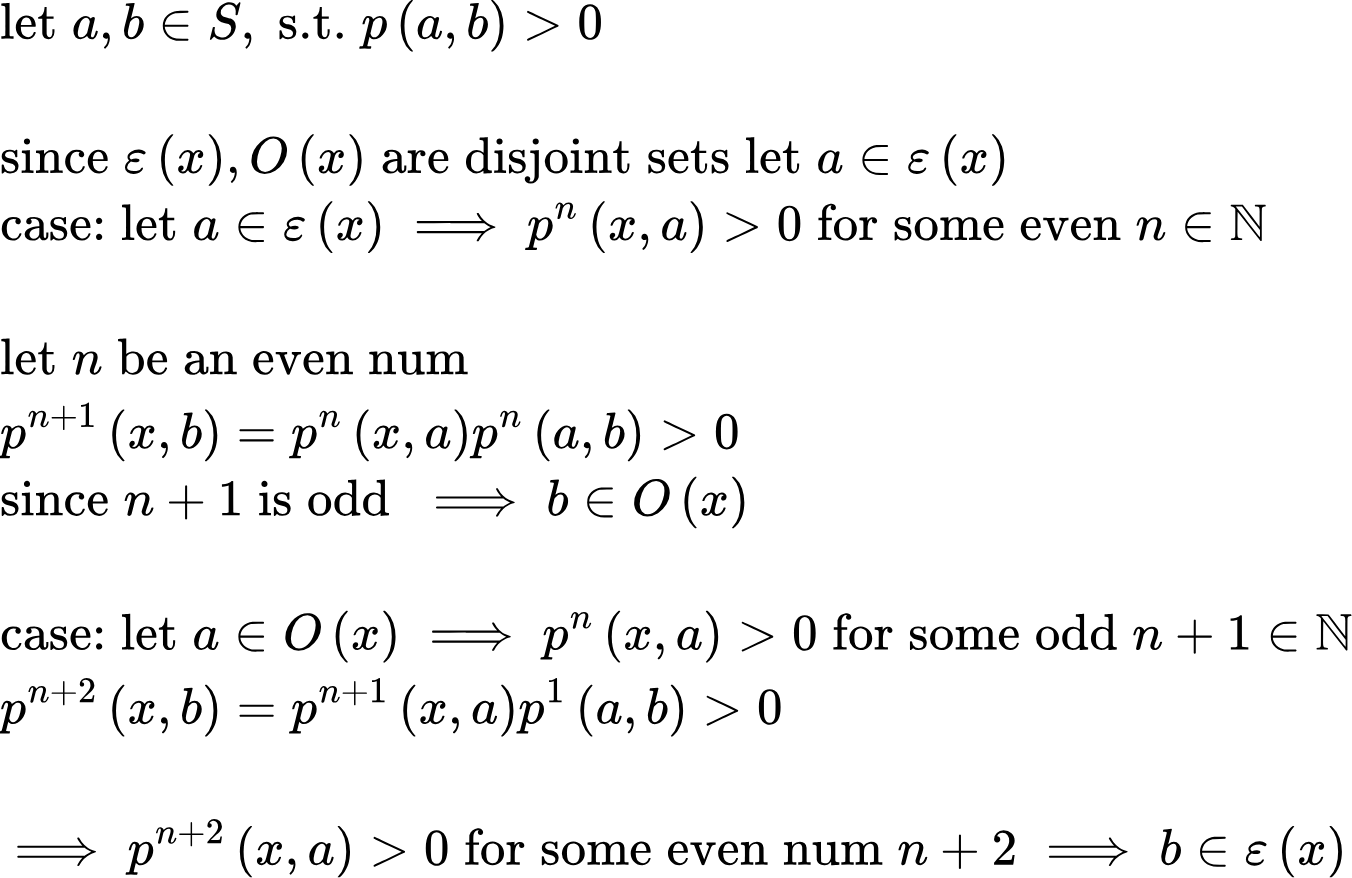


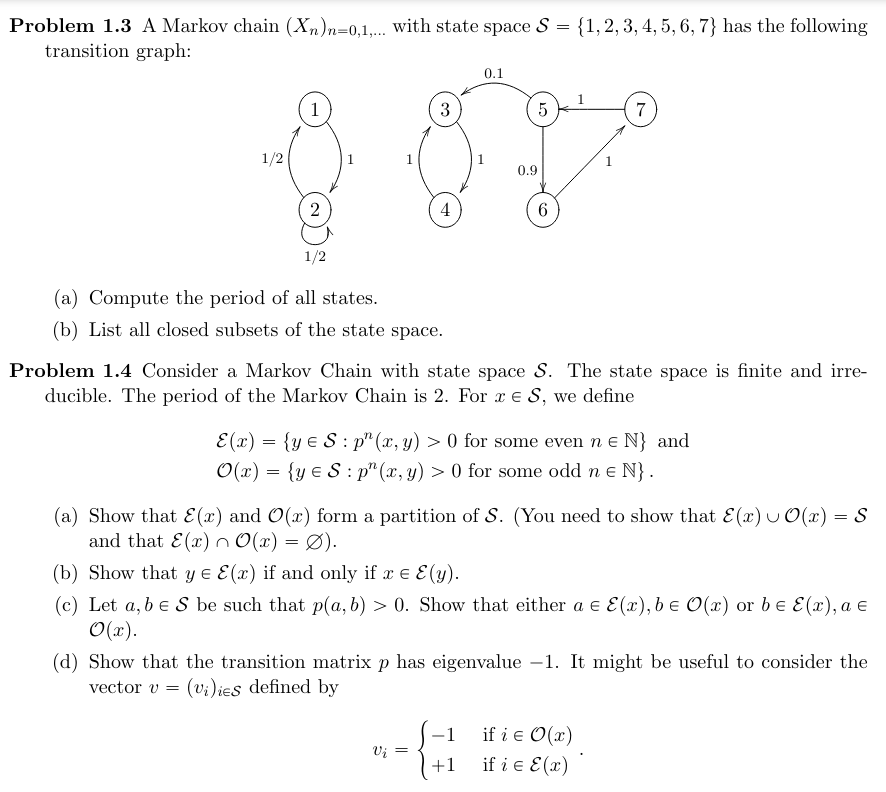


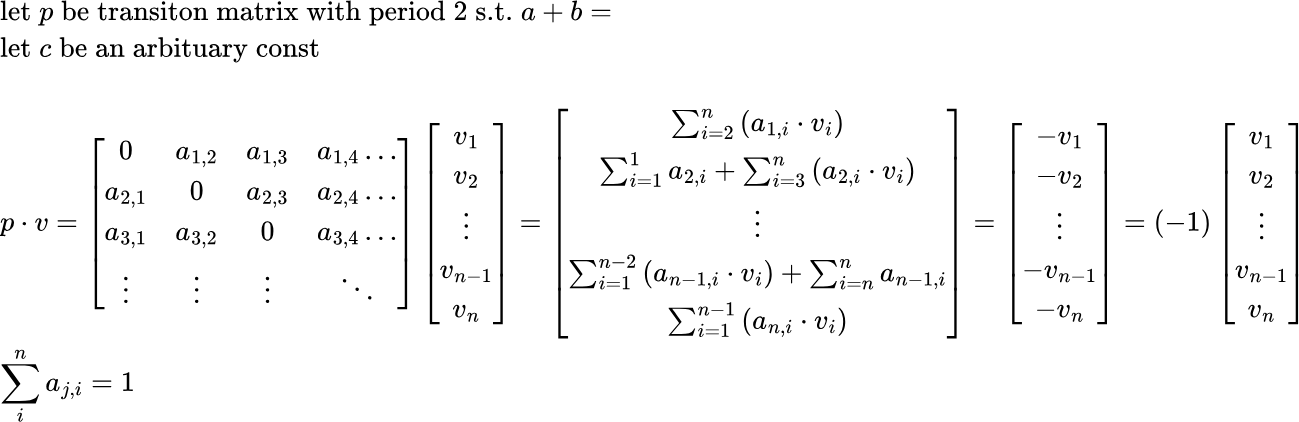


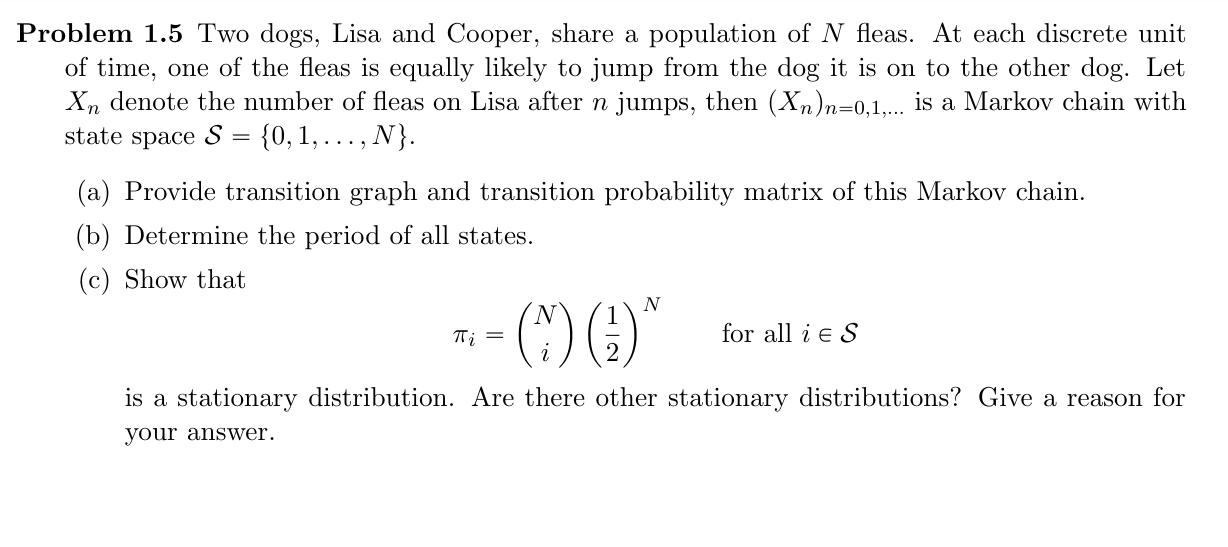


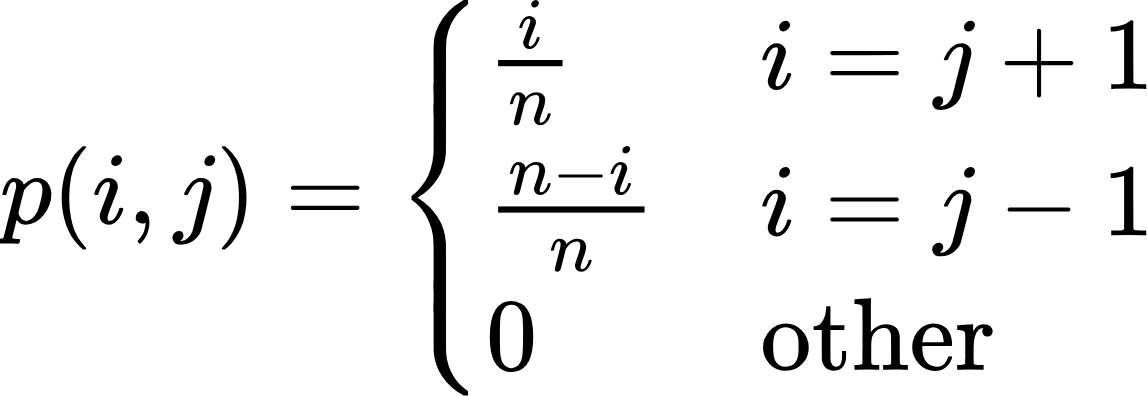


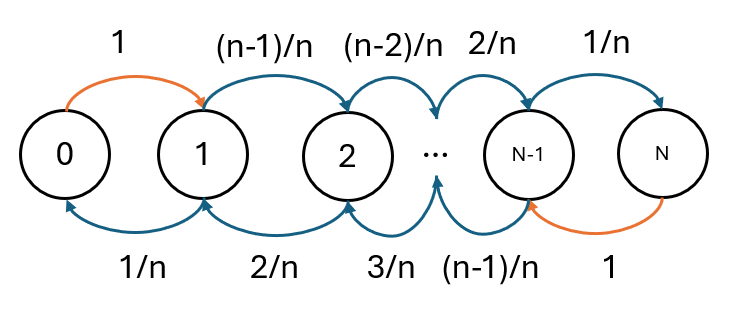


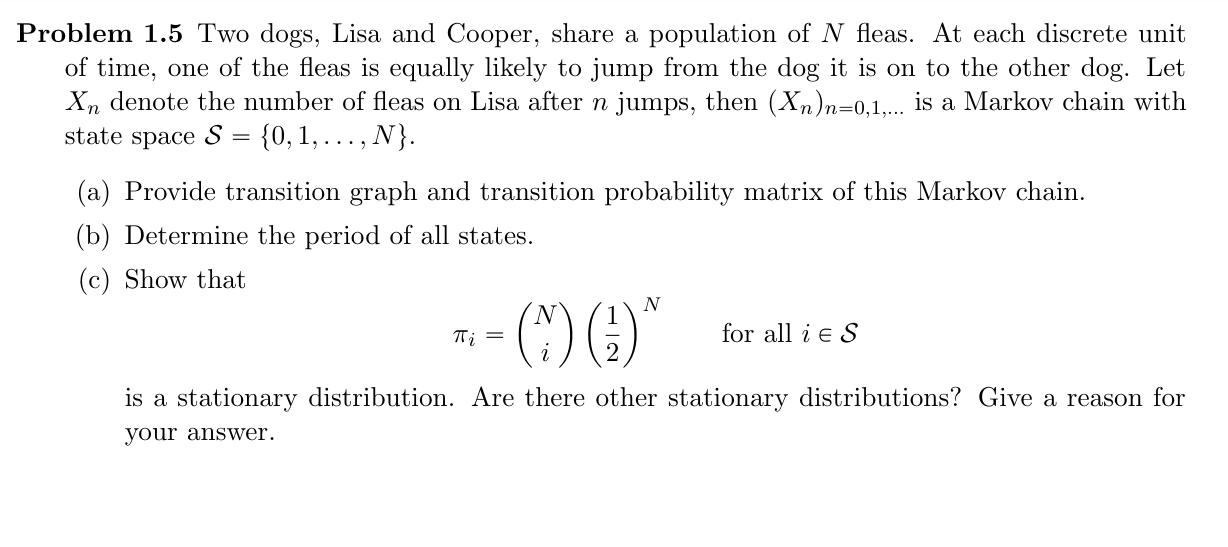


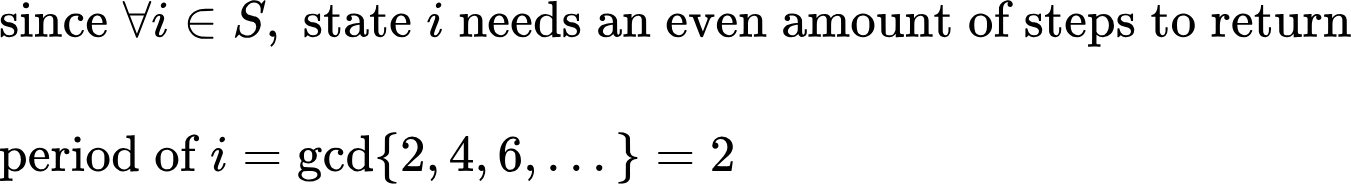


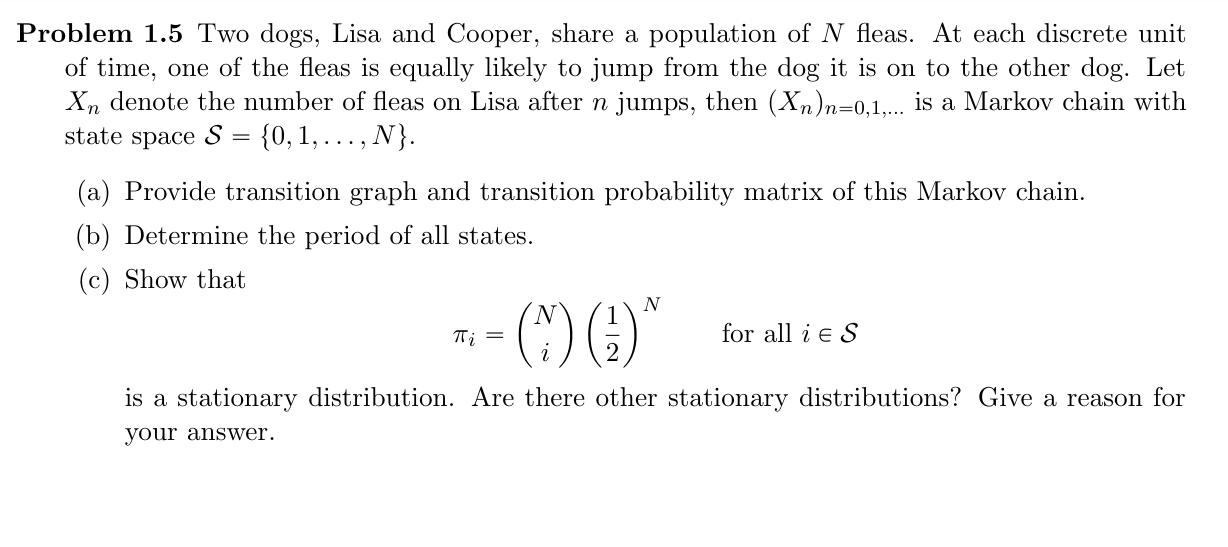


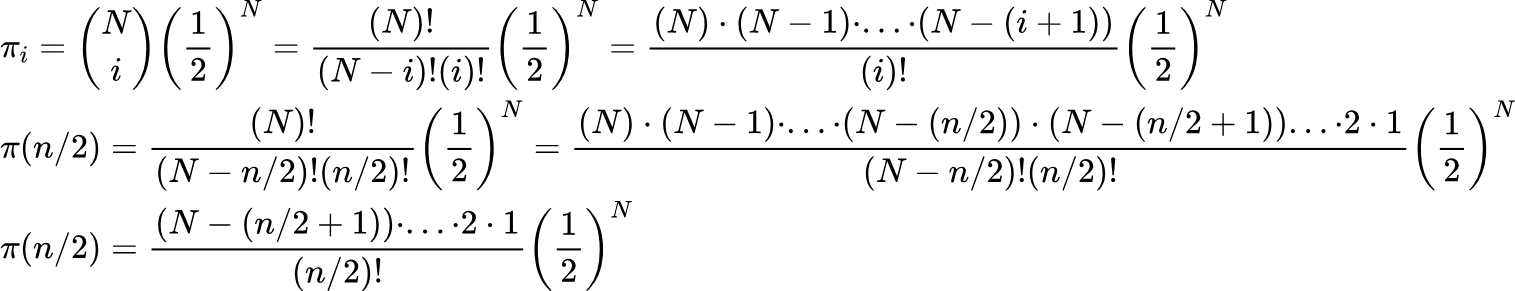












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